Name	Class

MATHS TEACHER HUB

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Algebraic proof

(9 – 1) Topic booklet

Higher

These questions have been collated from previous years GCSE Mathematics papers.

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser.

Total Marks

Instructions

•Use **black** ink or ball-point pen.
•**Fill in the boxes** at the top of this page with your name, centre number and candidate number.
•Answer **all** questions.
•Answer the questions in the spaces provided
 – *there may be more space than you need*.
•Diagrams are NOT accurately drawn, unless otherwise indicated.
•You must **show all your working out.**
•If the question is a **1H** question you are not allowed to use a calculator.
•If the question is a **2H** or a **3H** question, you may use a calculator to help you answer.

Information

•The marks for **each** question are shown in brackets
 – *use this as a guide as to how much time to spend on each question*.

Advice

•Read each question carefully before you start to answer it.
•Keep an eye on the time.
•Try to answer every question.
•Check your answers if you have time at the end.

Answer ALL questions
Write your answers in the space provided.
You must write down all the stages in your working.

12 Prove that the square of an odd number is always 1 more than a multiple of 4

June 2018 – Paper 1H **(Total for Question 12 is 4 marks)**

13 Given that n can be any integer such that $n > 1$, prove that $n^2 - n$ is never an odd number.

June 2019 – Paper 1H **(Total for Question 13 is 2 marks)**

14 Prove algebraically that

$(2n + 1)^2 - (2n + 1)$ is an even number

for all positive integer values of n.

Sample 1 – Paper 2H **(Total for Question 14 is 3 marks)**

15 Prove algebraically that the sum of the squares of any two consecutive even numbers is always a multiple of 4

November 2019 – Paper 3H **(Total for Question 15 is 3 marks)**

15 Show that $2^{40} - 1$ is the product of two consecutive odd numbers.

(2)

June 2023 – Paper 3H **(Total for Question 15 is 2 marks)**

15 Prove algebraically that the difference between the squares of any two consecutive odd numbers is always a multiple of 8

(Total for Question 15 is 3 marks)

16 (a) Prove that

$$(2m + 1)^2 - (2n - 1)^2 = 4(m + n)(m - n + 1)$$

(3)

Sophia says that the result in part (a) shows that the difference of the squares of any two odd numbers must be a multiple of 4

(b) Is Sophia correct?
You must give reasons for your answer.

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(1)

(Total for Question 16 is 4 marks)

16 n is an integer greater than 1

Prove algebraically that $n^2 - 2 - (n - 2)^2$ is always an even number.

May 2017 – Paper 1H **(Total for Question 16 is 4 marks)**

17 n is an integer.

Prove algebraically that the sum of $\frac{1}{2}n(n + 1)$ and $\frac{1}{2}(n + 1)(n + 2)$ is always a square number.

November 2017 – Paper 1H **(Total for Question 17 is 2 marks)**

17 The product of two consecutive positive integers is added to the larger of the two integers.

Prove that the result is always a square number.

(Total for Question 17 is 3 marks)

20 Prove algebraically that the difference between the squares of any two consecutive integers is equal to the sum of these two integers.

(Total for Question 20 is 4 marks)

22 Here are the first five terms of an arithmetic sequence.

7 13 19 25 31

Prove that the difference between the squares of any two terms of the sequence is always a multiple of 24

(Total for Question 22 is 6 marks)